## DECLARATION

This research project report is my original work and has not been presented for a degree in any other University.

Signature: ………………………………………. Date: …………………………

**Augustin DUSHIME**

**REG.N0: SC 384- CO10-4368/2013**

This research project report has been submitted for examination with our approval as University supervisors

Signature: ……………………………………… Date: ……………………………..

**Dr. Marcel NDENGO**

Signature: …………………………………….. Date: ………………………………..

**Dr. Joseph K. Mung'atu**

**DEDICATION**

To my beloved wife Adélphine TWAGIRAMALIYA and my daughter DUSHIME KEZA Curie,

I am grateful for your kind support, understanding and love.

**ACKNOWLEDGEMENTS**

I humbly thank God for providing enough graces to overcome all the barriers and come out with this project. I am using this opportunity to express my gratitude to everyone who supported me throughout the course of this project. I am sincerely grateful to them for sharing their truthful and illuminating views on a number of issues related to the project. I express my warm thanks to my two supervisors Dr. Joseph K. Mung'atu and Dr. Marcel NDENGO both from JKUAT for their support and guidance. I would also like to thank the management of MUHIMA District Hospital for supporting me with the information to use for this project. Special thanks to my family to whom I dedicate this work. Thanks to My Mother, sisters, brothers and all the people who provided me with the facilities being required and conductive conditions for my project.

**TABLE OF CONTENTS**

**LIST OF TABLES**

**LIST OF FIGURES**

## LIST OF ABBREVIATIONS

**FCFS**: First Come, First Served

**JKUAT:** Jomo Kenyata University of Agriculture and Technology

**LCFS**: Last Come, First Served

**OPD**: Outpatient Department

**SIRO**: Service in Random Order

**SPSS**: Statistical Package for Social Sciences

## ABSTRACT

The purpose of this project was to analyze time that patients can spend waiting for service in Muhima District hospital. The main objective for this research was to provide necessary information to policy makers aimed to contribute in wellbeing of population by reducing waiting time for service because in excessive cases, long queues can delay appropriate decision for a specific disease that can cause occurrence of death while patient still waiting for service. This project examined, first the waiting time of patients in outpatient department by using queuing model after calculating the mean number of arrivals per hour and the mean number of patients served per hour. Further results from questionnaire from staff were analyzed in order to know their opinions about the waiting time in outpatient department. The results showed that the system utilization factor is greater than one. This means that the queue grew without bound. There were a big number of patients waiting in the queue and they waited for a long time before being seen by a physician. The correlation analysis revealed a significant negative correlation between days and patient arrivals which means that there were many patients on Monday more than Friday. The reasons given by staff interviewed were the big number of patients visiting this department and the shortage of staff. To reduce the waiting time, we suggested increasing the number of physicians and nurses in outpatient department and strengthening the capacity building of health care providers. The hospital should develop a staffing plan and put more effort in the beginning of the week for efficient use of available resources.

# CHAPTER 1

## INTRODUCTION

### 1.1 Background

In order to respond to the demand of service on time and efficiently, many institutions use queuing models. However the use of queuing models is not widespread in hospitals. Regarding the efforts made by health facilities to prevent the harms that can be caused by delay of service, queuing models can contribute to allocate efficiently the available resources in their institutions.

There are queues as long as the subjects that request service, known as customers, are not immediately served when arrive at a service facility. In hospitals, patients are considered as customers and different departments such as laboratory, diagnostic imaging, pharmacy or outpatient department can be referred as service facilities.

A service facility can have one or more service stations where customers request service and each station can also have either one or more servers. For example, in laboratory department, a patient who needs a certain test can be requested to pass through two types of servers; cashier who receive money for the prescribed test and a nurse who takes the requested sample test.

A common characteristic of the majority of queuing models is that customers are discrete, and the number of customers waiting in the service facility is an integer value. Regarding the risks behind the queues in hospitals, the following questions can be asked:

(i) Why do queues form in the hospitals?

(ii) Why must patients wait to be served?

(iii) Which characteristics of the hospital system affect queuing and by how much?

This project attempted to provide answers to such questions in a typical case of MUHIMA Hospital located in Kigali city in Rwanda.

### 1.2 Statement of the Problem

The service facilities whose customers are patients vary generally in capacity and size, from small outpatient clinics to large, urban hospitals to referral hospitals. Regardless these differences, healthcare processes can be categorized based on how patients arrive, wait for service, obtain service, and then depart.

The healthcare processes also vary in complexity and extent, but they all deal with a set of both medical and non-medical activities and procedures that the patient must experience before getting the desired treatment. The servers in hospital queuing systems are the trained staff and equipment required for specific activities and procedures.

Almost all of us have waited for many hours, many days or many weeks to get an appointment with a medical doctor, and at arrival we are obliged to wait for a long time until being seen. In hospitals, it is not strange to get patients waiting for radiologist for imaging diagnosis and delays for surgery appointment.

Queues are everywhere, particularly in hospitals; lengthy queues are unfavorable for patients because delay in accessing needed services often cause prolonged pain and economic failure when patients are not able to work and potential deterioration of their medical conditions that can augment consequent treatment expenses and poor health outcomes. In excessive cases, long queues can delay appropriate decision for a specific disease that can cause occurrence of death while patient still wait for service.

Therefore, queuing has become a sign of incompetence of public hospitals in the world and Rwanda is not an exception. Decrease of waiting time of patients for healthcare service is one of the challenges facing the majority of hospitals. A few of the factors that is responsible for long waiting lines or delays in providing service are: lack of passion and commitment to work on the part of the hospital staff, overloading of available doctors, doctors attending to patients in more than one clinic etc. These put doctors under stress and tension, hence tends to dispose off a patient without in-depth probing or treatment, which often leads to patient dissatisfaction (Obamiro, 2010).

This project is based on the perceptive that most of these challenges can be managed by using queuing model to determine the waiting line performance such as: average arrival rate of patients, average service rate of patients, system utilization factor and the probability of a specific number of patients in the system. The resulting performance variables can be used by the policy makers to increase competence, improve the quality of patient care and reduce cost in hospital institutions as well.

## 1.3 Objectives

### 1.3.1 General Objective

The main objective of this project was to apply a queuing model for healthcare services in Muhima District Hospital.

### 1.3.2 Specific Objectives

This project had the following specific objectives:

1. To determine the mean number of arrivals per hour (λ) in Muhima District hospital.
2. To determine the mean number of patients served per hour (μ) in Muhima District hospital.
3. To compare the mean number of arrivals and the mean number of patients served per hour (λ and μ) in Muhima District hospital.
4. To determine the average time a patient spends waiting in the queue before being seen by a physician in Muhima District hospital.
5. To analyze the waiting line of patients in Muhima District hospital.

## 1.4 Research Questions

1. What is the mean number of arrivals per hour (λ)?
2. What is the mean number of patients served per hour (μ)?
3. What is the relationship between the mean number of arrivals and the mean number of patients served per hour (λ and μ)?
4. What is the average time a patient spends waiting in the queue before being seen by a physician?
5. What resources needed to reduce the length of queues in hospitals and increase patients' satisfaction?

## 1.5. Research Hypothesis

1. There is a long waiting time of patients before being seen by physicians in outpatient department of Muhima District hospital.

2. The system utilization factor is greater than one.

3. There is a significant negative correlation between days and patient arrivals in outpatient department of Muhima District hospital.

### 1.6 Justification

This research was conducted in order to fulfill the requirements for the award of the degree of Master of Science in Applied Statistics and it was benefit in different ways:

It will increase the knowledge of the student by relating the theories encountered from lectures to the real world of application. It will also contribute to increase patients' satisfaction in public health facilities. The decision makers in health system will benefit from this research by using results to develop their staffing plan. This project will serve as reference for other researchers in this field by filling the gaps encountered in present research.

### 1.7 Scope

Due to the financial and time constraints, this research has been conducted only to the patients visiting MUHIMA hospital in Outpatient Department (OPD) for consultation by a physician. A period of 36 days has been covered in which five days of each week from Monday to Friday were considered because they are the working days of the week, from 08:00 A.M to 12:00 AM and from 01:00 P.M to 05:00 P.M.

A questionnaire has been conducted to the nurses and physicians of the Outpatient Department to collect their opinions about causes and proposed solutions of queues.

## CHAPTER 2

## LITERATURE REVIEW

### 2.1 Introduction

Waiting in lines seems to be a component of a human daily life. Queues form when the demand for a service exceeds its supply. In hospitals, patients can wait a certain period of time (minutes, hours, days or months) to receive healthcare service. For many patients or customers, waiting in lines or queuing is annoying or negative experience. The disagreeable experience of waiting in line can often have a negative consequence on the rest of a customer's experience with a particular firm. The way in which managers address the waiting line issue is critical to the long term success of their firms.

Literature on queuing models indicates that waiting in line or queue causes problem to economic expenses to persons and institutions. Hospitals, banks, airline companies, industrialized firms etc., attempt to decrease the total waiting price, and the cost of service provided to their customers.

Therefore, speed of service is increasingly becoming a very important competitive parameter. Davis (2003) assert that providing ever-faster service, with the ultimate goal of having zero customer waiting time, has recently received managerial attention for several reasons. First, in the more highly developed countries, where standards of living are high, time becomes more valuable as a commodity and consequently, customers are less willing to wait for service. Second, this is a growing realization by organizations that the way they treat their customers today significantly impact on whether or not they will remain loyal customers tomorrow. Finally, advances in technology such as computers, internet etc., have provided firms with the ability to provide faster services.

For these reasons hospital managers and health providers are always finding way to deliver more rapidly services, believing that the waiting will negatively affect the organization performance evaluation. Cochran and Bharti (2006) also argue that higher operational efficiency of the hospital is likely to help to control the cost of medical services and consequently to provide more affordable care and improve access to the public. Researchers have argued that service waits can be controlled by two techniques: operations management or perceptions management (Hall, 2006).

The operation management feature deals with the organization of how customers (patients), queues and servers can be coordinated towards the goal of rendering efficient service at the minimum cost. The act of waiting has significant impact on patients' satisfaction. The amount of time customers must spend waiting can significantly influence their satisfaction. Additionally, research has demonstrated that customer satisfaction is affected not just by waiting time but also by customer expectations or attribution of the causes for the waiting. Consequently, one of the issues in queue management is not only the actual amount of time the customer has to wait, but also the customer's perceptions of that wait. Clearly, there are two approaches to increasing customer satisfaction with regard to waiting time: through decreasing actual waiting time, as well as through enhancing customer's waiting experience (Singh, 2011).

## 2.2 Theoretical Review

### 2.2.1 Queuing Theory

**Queuing theory** is the mathematical study of waiting lines, or queues (Sundarapandian, 2009). In queuing theory a model is constructed so that queue lengths and waiting times can be predicted (Sundarapandian, 2009). Queuing theory is generally considered as branch of operations research because the results are often used when making business decisions about the resources needed to provide a service. Queuing theory has its origins in research by Agner Krarup Erlang when he created models to describe the Copenhagen telephone exchange (Sundarapandian,2009). The ideas have since seen applications including telecommunication, traffic engineering, computing and the design of factories, shops, offices and hospitals (Schlechter, 2009).

### i) Essential components to describe a phenomenon of waiting line

The following components are essential to describe a phenomenon of waiting line: the population source, the arrival, queues, queue discipline, service mechanism, departure or exit.

#### a. Population source

The population source serves as where arrivals are generated. Arrivals of patients at the hospital may be drawn from either a finite or an infinite population. A finite population source refers to the limited size of the customer pool. Alternatively, an infinite source is forever.

#### b. Queue discipline

The queue discipline is the sequence in which customers or patients are processed or served.

The most common discipline is first come, first served (FCFS). Other disciplines include last come, first served (LCFS) and service in random order (SIRO). Customers may also be selected from the queue based on some order of priority (Taha, 2005).

**c. Service mechanism**

The service mechanism describes how the customer is served. It includes the number of servers and the duration of the service time-both of which may vary greatly and in a random fashion. The number of lines and servers determines the choice of service facility structures. The common service facility structures are: single-channel, single-phase; single-channel, multiphase; multi-channel, single phase and multi-channel, multiphase.

**d. Departure or exit**

The departure or exit occurs when a customer is served. The two possible exit scenarios as mentioned by Davis (2003) are: (a) the customer may return to the source population and immediately become a competing candidate for service again; (b) there may be a low probability of re-service.

### ii) Birth And Death Process Queuing Models

A number of important queuing theory models fit the birth-and-death process. A queuing system based on the birth-and–death process is in state $E_n$ at time t if the number of customers is then $n$, that is, $N(t)=n$. A birth is a customer arrival, and a death occurs when a customer leaves the system after completing service. Thus, given the birth rates $\{\lambda_n\}$ and death rate $\{\mu_n\}$, and assuming that

$$S=1 + C_1 + C_2 + C_3 + \ldots < \infty \tag{1}$$

Where

$$C_n = \frac{\lambda_0 \lambda_1 \cdots \lambda_{n-1}}{\mu_1 \mu_2 \cdots \mu_n}, \qquad n = 1, 2, 3, \ldots, \tag{2}$$

We calculate

$$P_0 = 1/S \tag{3}$$

and $$P_n = P[N=n] = C_n P_0, \qquad n = 1, 2, 3, \ldots \tag{4}$$

From the probabilities calculated by (4) we can generate measures of queuing system performance (Nosek, 2001).

**iii) M/M/1 Queuing System**


**Figure 2.1: M/M/1 Queuing System.**

This model assume a random (Poisson) arrival pattern and a random (exponential) service time distribution. The arrival rate does not depend upon the number of customers in the system and the probability of an arrival in a time interval of length h>0 is given by

$$e^{-ªh}(ªh) = ªh(1 - ªh + \frac{(ªh)^2}{2!} - ...)$$
$$= ªh - (ªh)^2 - \frac{(ªh)^3}{2!} - ... + (-1)^{n+1}\frac{(ªh)^n}{(n-1)!} + ... \quad (5)$$
$$= ªh + o(h)$$

Thus, we have

$$ª_n = ª \qquad n= 0, 1, 2, \dots \quad (6)$$

By hypothesis, the service time distribution is given by

$$W_s(t) = P[s \le t] = 1 - e^{-«t}, \quad t \ge 0. \quad (7)$$

Then, when a customer is receiving service, the probability of a service completion (death) is a short time interval, *h*, is given by

$$1 - e^{-«h} = 1 - (1 - «h + \frac{(«h)^2}{2!} - ...) = «h + o(h). \quad (8)$$

(Here we have used the memoryless property of the exponential distribution in neglecting the service already completed.) (Little, 1961).

Thus,

$$«_n = «, \qquad n=1, 2, 3, \dots. \quad (9)$$

Thus the state-transition diagram for the M/M/1 queuing system is given by Figure 2 and therefore, by (1), since $\lambda/\mu=\rho$, and each $C_n$ is equal to $\rho^n$,

$$S=1+\rho+\rho^2+...+\rho^n+...=1/(1-\rho). \tag{10}$$

Hence,

$$p_n=P[N=n]=(1-\rho)\rho^n, \qquad \text{n=0, 1, 2, …} \tag{11}$$

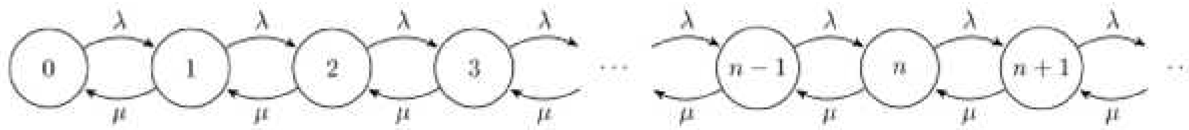


**Figure 2.2: State-transition diagram of the M/M/1 queuing system.**

But (11) is the pmf for a geometric random variable, that is, N has a geometric distribution with $p=1-\rho$ and $q=\rho$. Hence,

$$L=E[N]=q/p=\rho/(1-\rho), \tag{12}$$

and

$$\sigma_N^2=\rho/(1-\rho)^2. \tag{13}$$

By Little's formula,

$$W=E[q]=W-E[s]=\rho E[s]/(1-\rho), \tag{14}$$

since $\rho=\lambda E[s]$.

Now,

$$W_q=E[q]=W-E[s]=\rho E[s]/(1-\rho). \tag{15}$$

Applying Little's formula, again, gives,

$$L_q=E[N_q]=\lambda W_q=\rho^2/(1-\rho). \tag{16}$$

By (11) we calculate

P[server is busy]=1-P[N=0]=1-(1-° )=° .

By the law of large numbers this probability can be interpreted as the fraction of time that the sever is busy; it is appropriate to call ° the "server utilization".

We now have the four parameters most commonly used to measure the performance of a queuing system, $W$, Wq, L and Lq as well as the pmf, $p_n$, of the number in the system.( Allen, 1978)

**iv) The M/M/1/K Queuing System**

The M/M/1/K system is a more accurate model of this type of system in which a limit of K customers is allowed in the system. When the system contains K customers, arriving customers are tuned away. Figure 3 is the state-transition diagram for this model. Thus, a birth-and-death process, the coefficients are (Parzen, 1962)

$$\text{a}_n = \begin{cases} \text{a} & \text{for } n=0, 1, 2, ..., k-1 \\ 0 & \text{for } n \geq K \end{cases} \quad (17)$$

and

$$«_n = \begin{cases} « & \text{for } n=0, 1, 2, ..., K \\ 0 & \text{for } n>K \end{cases} \quad (18)$$

This gives the steady state probabilities

$$P_n = \left(\frac{\text{a}}{«}\right)^n P_0 = u^n P_0 \quad \text{for } n=0, 1, 2, ...,K, \quad (19)$$

Where

$u = \text{a}E[s] = \text{a} / «.$

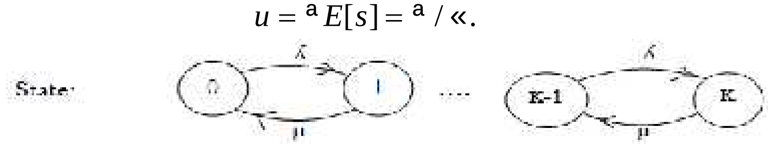

**Figure 2.3: State-Transition diagram for the M/M/1/K queuing system.**

Since

$$1 = p_0 + p_1 + ... + p_K = p_0 \sum_{n=0}^{K} u^n = \left( \frac{1-u^{K+1}}{1-u} \right) p_0,$$

if ª ≠ «, we have,

$$p_0 = (1-u) \big/ \left(1-u^{K+1}\right) \qquad (20)$$

Since there are never more than K customers in the system, the system reaches a steady state for all values of ª and « . That is, we need not assume that ª < « for the system to achieve a steady state. If ª = « then ° = 1 and

$$p_0 = 1 \big/ (K+1) = p_n \qquad \text{for n=1, 2, …, K}$$

Thus the steady state probabilities are

$$p_n = \begin{cases} \dfrac{(1-u)u^n}{1-u^{K+1}} & \text{for ª ≠ «, n=0, 1, ....,K} \\ \dfrac{1}{\text{K+1}} & \text{for ª=«, n=0, 1, ..., K} \end{cases} \qquad (21)$$

It should be noted that, if ª < «, as $K \to \infty$, each $p_n$ in (20) approaches the value in (11), as it should (Gross, 1974).

If ª ≠ « then

$$L = E[N] = \sum_{n=1}^{K} np_n = \sum_{n=1}^{K} \left(\frac{1-u}{1-u^{K+1}}\right)\left(nu^n\right) = \left(\frac{1-u}{1-u^{K+1}}\right) u \sum_{n=1}^{K} nu^{n-1}$$

$$= \left(\frac{1-u}{1-u^{K+1}}\right) u \sum_{n=1}^{K} \frac{du^n}{du} = \left(\frac{1-u}{1-u^{K+1}}\right) u \frac{d}{du} \sum_{n=0}^{K} u^n$$

Continuing calculations in the same way we get

$$L = \frac{u}{1-u} - \frac{(K+1)u^{K+1}}{1-u^{K+1}}. \tag{22}$$

Thus if ª < « , the expected number in the system, L, is always less than for the unlimited queue length case (where L is $u/(1-u)$ ).

If ª = « then $u = 1$ and

$$L = \sum_{n=1}^{K} np_n = \frac{1}{K+1}(1+2+\ldots+K) = \frac{K(K+1)}{2(K+1)} = \frac{K}{2}. \tag{23}$$

Thus (21) and (4) can be summarized by

$$L = \begin{cases} \dfrac{u}{1-u} - \dfrac{(K+1)u^{K+1}}{1-u^{K+1}} & \text{if ª} \neq \text{«} \\[2ex] \dfrac{K}{2} & \text{if ª} = \text{«} \end{cases} \tag{24}$$

In either case,

$$L_q = L - (1 - p_0) \tag{25}$$

because

$$E[N_s] = P[N=0]E[N_s \mid N=0] + P[N>0]E[N_s \mid N>0]$$

$$= p_0 * 0 + (1-p_0) * 1 = 1 - p_0.$$

All the traffic reaching the system does not enter the system because customers are not allowed admission when there are K customers in the system, that is, with probability $p_K$. Thus, if $\lambda_a$ is the average rate of customers into the system, (Capasso, 2008).

$$\lambda_a = \lambda\left(1 - p_k\right). \tag{26}$$

We can then apply Little's formula to obtain

$$W = E[w] = L/\lambda_a, \tag{27}$$

and

$$W = E[q] = L_q/\lambda_a, \tag{28}$$

The true server utilization, $\rho$, which is the probability that the server is busy, is given by

$$\rho = \lambda_a E[s] = \lambda(1 - p_K)E[s] = (1 - p_K)u. \tag{29}$$

v) **The M/M/c Queuing System**

For this model we assume random (exponential) interarrival and service times with C identical servers (Asmussen, 2003). This system can be modeled as a birth-and-death process with the coefficients

$$\lambda_n = \lambda, \qquad \text{n=0, 1, 2, ...,} \tag{30}$$

and

$$\mu_n = \begin{cases} n\mu, & \text{n=1,2,3,...,c} \\ c\mu, & \text{n=c, c+1, ...} \end{cases} \tag{31}$$

The state-transition diagram is shown in Figure. (4).

Thus, by (2), with $u = \lambda/\mu$ and $\rho = u/c$,

$$S = \frac{1}{p_0} = 1 + u + \frac{u^2}{2!} + \ldots + \frac{u^{c-1}}{(c-1)!} + \frac{u^c}{c!}\left(1 + \frac{u}{c} + \left(\frac{u}{c}\right)^2 + \ldots\right)$$

$$= \sum_{n=0}^{c-1} \frac{u^n}{n!} + \frac{u^c}{c!} \sum_{n=0}^{\infty} \circ^n = \sum_{n=0}^{c-1} \frac{u^n}{n!} + \frac{u^c}{c!(1-\circ)}. \qquad (32)$$

Hence, $p_0 = \left[\sum_{n=0}^{c-1} \frac{u^n}{n!} + \frac{u^c}{c!(1-\circ)}\right]^{-1}$ (33)

State:

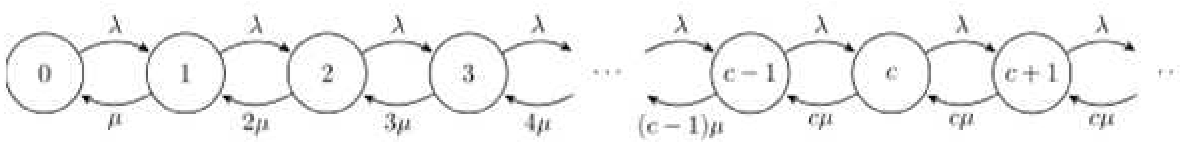


**Figure 2.4: State-transition diagram for $M/M/c$ queuing system.**

and

$$p_n = \begin{cases} \frac{u^n}{n!} p_0 & \text{if n=0, 1, 2, ...,c.} \\ \frac{u^n}{c!c^{n-c}} p_0 & \text{if n} \geq \text{c} \end{cases} \qquad (34)$$

We will now derive the primary measures of system performance, $L_q$, $W_q$, W and L.

By definition, (Medhi, 2002)

$$L_q = E\left[N_q\right] = \sum_{n=c}^{\infty} (n-c) p_n = \sum_{k=0}^{\infty} k\, p_{c+k} = \sum_{k=0}^{\infty} k \frac{u^c}{c!} \circ^k p_0 = p_0 \frac{u^c}{c!} \sum_{k=0}^{\infty} k \circ^k$$

$$= p_0 \frac{u^c}{c!}\left\{0 + 1\circ + 2\circ^2 + 3\circ^3 + \ldots\right\} = p_0 \frac{u^c}{c!} \circ \frac{d}{d\circ}\left\{1 + \circ + \circ^2 + \ldots\right\}$$

$$= p_0 \frac{u^c}{c!} \circ \frac{d}{d\circ}\left(\frac{1}{1-\circ}\right) = \frac{p_0 u^c \circ}{c!(1-\circ)^2}. \qquad (35)$$

Equation (35) can also be written as

$$L_q = \frac{\circ^{c+1}}{(c-1)!(c-\circ)^2} * p_0 \qquad (36)$$

Having computed $L_q$ by the formula (35), we can calculate

$$W_q = L_q / \text{ª} \qquad (37)$$

$$W = W_q + E[s] = W_q + (1/\text{«}), \qquad (38)$$

and

$$L = \text{ª} W \qquad (39)$$

From Equation (37), (39) can be expressed as

$$L = L_q + \text{ª}/\text{«} \qquad (40)$$

While we have the pmf of the number in the system, N, and the expected values of the primary random variables, it is useful to have the distribution functions of $w$ and $q$. We will drive $W_q(.)$ and state the formula for $W(.)$.

First, we note that

$$W_q(0) = P[q=0] = P[N \le c-1] = \sum_{n=0}^{c-1} p_n = p_0 \sum_{n=0}^{c-1} \frac{u^n}{n!}. \qquad (41)$$

But, by (33)

$$p_0\left(\sum_{n=0}^{c-1} \frac{u^n}{n!}\right) + \frac{p_0 u^c}{c!(1-\circ)} = 1$$

Or

$$p_0\left(\sum_{n=0}^{c-1} \frac{u^n}{n!}\right) = 1 - \frac{p_0 u^c}{c!(1-\circ)}.$$

Therefore, we have

$$W_q(0) = 1 - \frac{p_0 u^c}{c!(1-°)} = 1 - \frac{p_c}{1-°}. \tag{42}$$

Now, suppose $N = n \geq c$ when a customer arrives. All *c* servers are busy so the time between service completions has an exponential distribution with average value $1/c«$ .

There are *c* customers receiving service and *n-c* customers waiting in the queue. Therefore, the new arrival must wait for $n-c+1$ service completions before receiving service. (If *n=c*, so no customer is waiting, the new arrival must wait for one service completion. If *n=c+1*, two service completions are required, etc.)

Hence, the waiting time in queue is the sum of *n-c+1* independent exponential random variables each with mean $1/c«$ ; that is it is gamma with parameters $n-c+1$ and $c«$ . Hence, if t>0, we can write, by (18), since $\Gamma(n-c+1) = (n-c)!,$ that (Baccelli, 2003)

$$
\begin{aligned}
W_q(t) &= W_q(0) + \sum_{N=c}^{\infty} p_n P\left[q \le t \mid N = n\right] \\
&= W_q(0) + \sum_{N=c}^{\infty} p_0 \frac{u^n}{c!c^{n-c}} \int_0^t \frac{c\mu(c\mu x)^{n-c}}{(n-c)!} e^{-c\mu x} dx \\
&= W_q(0) + p_0 \frac{u^c}{(c-1)!} \int_0^t \mu e^{-c\mu x} \left( \sum_{n=c}^{\infty} \frac{(\mu u x)^{n-c}}{(n-c)!} \right) dx \\
&= W_q(0) + \frac{p_0 u^c}{(c-1)!} \int_0^t \mu e^{-c\mu x} e^{-\mu u x} dx \\
&= W_q(0) + \frac{p_0 u^c}{(c-1)!} \int_0^t \mu e^{-\mu x (c-u)} dx \\
&= W_q(0) + \frac{p_0 u^c}{(c-u)(c-1)!} \left(1 - e^{-\mu t (c-u)}\right) \\
&= 1 - \frac{p_0 u^c}{(c-u)(c-1)!} + \frac{p_0 u^c}{(c-u)(c-1)!} \left(1 - e^{-\mu t (c-u)}\right) \\
&= 1 - \frac{p_0 u^c}{(1-\rho)c!} e^{-\mu t (c-u)} \\
&= 1 - \frac{p_c}{(1-\rho)} e^{-\mu t (c-u)} = 1 - \frac{p_c}{(1-\rho)} e^{-c\mu t (c-\rho)}.
\end{aligned} \tag{43}
$$

The quantity $p_c/(1-\rho)$ is an interesting quantity; in fact it is the probability that an arriving customer must wait; it is known as *Erlang's C formula* or Erlang's delay formula and written

$$
C(c,u) = \frac{u^c}{c!(1-\rho)} p_0 = \frac{u^c/c!}{(1-\rho)\left[\left(\sum_{n=0}^{c-1} \frac{u^n}{n!}\right) + \frac{u^c}{c!(1-\rho)}\right]}, \tag{44}
$$

Where, of course, $u = \lambda/\mu$ and $\rho = u/c$.

To see that Erlang's C formula, (44), does give the probability that an arriving customer must wait we note that this probability is

$$
\sum_{n=c}^{\infty} p_n = 1 - \sum_{n=0}^{c-1} p_n = 1 - W_q(0) = \frac{p_c}{1-\rho}, \quad \text{by (42).}
$$

Hence (41) can be written as

$$W_q(t) = 1 - P[\text{arriving customer must queue}]e^{-c« (1- ° )}$$

$$= 1 - C(c,u)e^{-c«t(1- ° )}, \qquad t \geq 0 \tag{45}$$

Formula (43) can be used to calculate the $r^{th}$ percentile value of $q$, ®q (r).

The distribution function W(.) for the waiting time is given by

$$W(t) = 1 + \frac{(u - c + W_q(0))}{c - 1 - u} e^{-«t} + \frac{C(c,u)}{c - 1 - u} e^{-c«t(1- ° )}, \qquad \text{if u} \neq c - 1 \tag{46}$$

and by

$$W(t) = 1 - [1 + C(c,u)«t]e^{-«t}, \qquad \text{if u} = c - 1 \tag{47}$$

**vi) The M/M/c/c Queuing System**

This system is sometimes called the *M/M/C loss system* because customers who arrive when all servers are busy are not allowed to wait for service and are lost. The state-transition diagram is given in Figure 5.

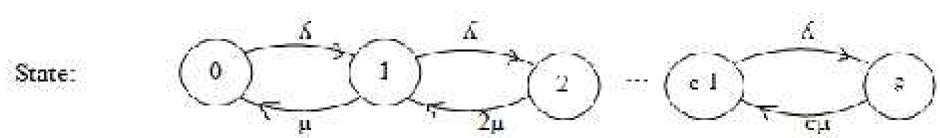


**Figure 2.5: State-transition diagram for $M/M/c/c$ queuing system.**

From the diagram we see that

$$C_n = u^n / n!, \qquad \text{n=1, 2, ..., c,} \tag{48}$$

Where, as usual, $u = ª E[s] = ª/«$, and

$$S = 1/p_0 = 1 + u + u^2/2! + \ldots + u^c/c!. \tag{49}$$

Thus

$$p_n = \frac{u^n / n!}{1 + u + u^2/2! + \ldots + u^c/c!} = B(c,u). \quad \text{n=0, 1, 2, ..., c.} \tag{50}$$

The distribution given by (50) is called "truncated Poison distribution," for obvious reasons (Puhalskii, 1962). In particular, the probability that all servers are busy, so that an arriving customer is lost, is

$$p_n = \frac{u^n / n!}{1 + u + u^2/2! + \ldots + u^c/c!} = B(c,u). \tag{51}$$

$B(c,u)$ is called "Erlang's *B* formula" or "Erlang's loss formula" in honor of It's discoverer, A.K.Erlang. Just as with M/M/1/K model the actual average arrival rate into the system, $ª_a$, is less than ª because some arrivals are tuned away.

We must have

$$ª_a = ª\left(1 - B(c,u)\right). \tag{52}$$

Since no customers are allowed to wait, $W_q$ and $\mathrm{L_q}$ are zero. However,

$$L = E[N] = \sum_{n=0}^{c} np_n = p_0 \sum_{n=1}^{c} n\frac{u^n}{n!} = u\,p_0 \sum_{n=0}^{c-1} \frac{u^n}{n!} = u(1 - B(c,u)). \tag{53}$$

By Little's formula,

$$W = E[w] = L/ª_a = E[s]. \tag{54}$$

Of course (54) is obvious because there is no waiting. Thus *w* has the same distribution as *s* (Giambene, 2005).

This means that

$$W(t) = P[w \le t] = 1 - e^{-«t} = 1 - e^{-t/E(s)}. \tag{55}$$

### vii) M/M/∞ Queuing System

No real life queuing system can have an infinite number of servers; what is meant, here, is that a server is immediately provided for each arriving customer. The state-transition diagram for this model is shown in Figure 6. We can read off from the figure that (Sztrik, 2000)

$$C_n = u^n / n!, \qquad n = 1, 2, 3, \ldots$$

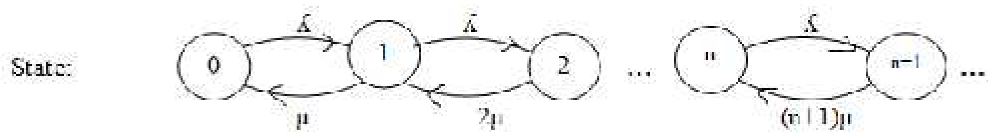


**Figure 2.6: State-transition diagram for $M/M/\infty$ queuing system**

so that

$$S = 1/p_0 = \sum_{n=0}^{\infty} u^n / n! = e^u.$$

Hence,

$$p_n = e^{-u}(u^n / n!), n = 0, 1, 2, \ldots \qquad (56)$$

that is, N has a Poison distribution! The fact that $p_n$ has a Poisson distribution tells us that $L = E[N] = u$ is the average number of busy servers, with $Var[N] = u$. The $M/M/\infty$ queuing model can be used to estimate the number of lines in use in a large communication network or as a gross estimate of values in an $M/M/c$ or $M/M/c/c$ queuing system.

### 2.2.2 Application of Queuing Theory in Outpatient Departments

The health system's ability to deliver safe, efficient and smooth services to the patients did not receive much attention until mid 1990's. Several key reimbursement changes, increasing critiques and cost pressure on the system and increasing demand of quality and efficacy from highly aware and educated patients due to advances in technology and telecommunications, have started putting more pressure on the healthcare managers to respond to these concerns (Singh, 2011).

Queuing theory manages patient flow through the system. If patient flow is good, patients flow like a river, meaning that each stage is completed with minimal delay, when the system is broken, patients accumulate like a reservoir (Hall, 2006). Healthcare systems resemble any complex queuing network in that delay can be reduced through: (1) Synchronization of work among service stages, (ii). Scheduling of resources (e.g. doctors and nurses) to match patterns of arrival and, (iii) constant system monitoring (e.g. treating number of patients waiting by location, diagnostic grouping) linked to immediate actions (Hall, 1991). Recently, application of stochastic methods has increased in analyzing clinical problems (Kandemir, 2007).

## CHAPTER 3

## METHODOLOGY

### 3.1 Introduction

According to the dictionary the Petit Larousse illustrated (2008), the methodology *is "The systematic study by observation of the scientific practice, the basic principles and methods of research that it uses".* The methodological framework of our work described all the methods and instruments used to collect the information we need as well as the procedure of the analysis and interpretation of the information gathered.

### 3.2 Research Design

A **research design** is a systematic plan to study a scientific problem. (Gorard, 2013)

In this study, the patients are coming from infinite population and the system was enough to receive all the patients coming in Outpatient consultation.

There were also three consultation rooms (3 servers) to receive patients (customers).

From above conditions the model to be used is $M/M/c$ : FCFS/∞/∞ where;

M=Markovian (or poisson) arrivals and exponential service time.

$c =$ Multi-server; where in our case $c$ is equal to three physicians working in outpatient department.

FCFS = First come, first served;

∞ = Infinite system limit;

∞ = Infinite source limit.

For the purpose of modeling, the arrivals (n) are the outpatients. As each reaches the hospital, he/she books for service. If service is rendered immediately he/she leaves the hospital or otherwise joins the queue. The doctors are the servers (*c*).

The arrival rate, service time and number of servers were the data used for the study that have been collected using observation method. The data collection covered a period of 36 days in which five days of each week from Monday to Friday were considered because they are the working days of the week.

### 3.3 **Population of Inquiry**

Depelteau. F. (2000, p. 213) defines the population as being" *a set of all individuals who have precise characteristics in relationship with the objectives".* For our case, the population of inquiry concerned the patients who are coming for consultation in Outpatient department in Muhima hospital.

### 3.4 Sampling Frame

In statistics, a sampling frame is the source material or device from which a sample is drawn. It is a list of all those within a population who can be sampled, and may include individuals, households or institutions (Carl, 2003). For our case, the sampling frame concerned the patients who were coming for consultation in Outpatient department every day in Muhima hospital.

However, given that the patients arrive randomly and we are most interested by the period between two successive patients, we have taken our sampling frame as number of hours in a year to collect arrival and service data.

### 3.5 Sample and Sampling Technique

Because of the inability to achieve individually all units of our statistical universe, we have carried out the sampling technique. According to Hammersley (2002), sampling is the fact of choosing a limited number of individuals, objects, events which the observation allows to draw conclusions applicable to the whole population from which the choice has been made. To determine the sample of our study, we have used the following formula:

$$n = \frac{z^2 p(1-p)}{E^2} \tag{57}$$

Where: Z = Z-score (=1.96 for 95% confidence level)

p = Healthcare service utilization rate in Muhima hospital which corresponds to 86%

E = Margin of Error (Confidence Interval); in our case we have decided to use 0.04

The table below shows different values used in calculation and the corresponding sample size n.

**Table 3.1: Sample size calculations**

| Z | p | 1-p | E | N |
|---|---|---|---|---|
| 1.96 | 0.86 | 0.14 | 0.04 | 289 hours |

$$n = \frac{(1.96)^2 0.86 * (1-0.86)}{(0.04)^2} = 289 hours \tag{58}$$

Here we obtain a sample size of 289 hours that correspond to 36 days if we consider 8 working hours per day. We have also selected a total number of 16 staff to interview on the prepared questionnaire without sampling because this number was small and it was not necessary to take a sample from them.

### 3.6 Instruments

In our work, different documents have been used such as books, reports and electronic sources. All these documents helped us to make the conceptual and theoretical framework of our work as well as to analyze the data and interpret the results. Also, we have used a register to record discrete time for patient arrival and service. For collection of staff opinions, a questionnaire has been used.

### 3.7 Data Collection Procedure

In this project the observation technique has been used where we registered the time when every patient enters in outpatient department and a time when he/she comes out from outpatient department. This helped to draw a table used in estimating the average number of patients entered in the system and average number of patients served in one hour. From this we have estimated the remaining performance parameters of the system. These data have been collected for a period of 36 days from Monday to Friday, from 08:00 A.M to 12:00 and from 01:00 P.M to 05:00 P.M. A questionnaire has been used to collect staff opinions about causes and proposed solutions of queues.

### 3.8 Data Processing and Analysis

For analysis of our data and interpretation of the results, different computer tools have been used especially Microsoft Excel and SPSS. The data collected using observation technique has been entered in Excel spread sheet for cleaning and convert the recorded time in interval time and then imported in SPSS for analysis where descriptive statistics and significance test have been carried out as well as estimation of different performance parameters describing the behavior of the system.

The data from questionnaire has been directly entered in a designed SPSS sheet for cleaning and analysis. The figures and tables were interpreted in scope predefined objectives in order to make data meaningful and come out with conclusions and recommendations.

The system performance parameters used in this study were defined as follows:

**λ**: Arrival rate of patients at outpatient department per hour;

**μ**: Service rate ( Length of stay) of patients at outpatient department per hour;

**C**: Number of doctors (servers) working in outpatient department for consultation.

In this model, there are three parallel physicians

° : Outpatient system utilization factor = λ/Cμ,

**Lq**: Average number of patients at outpatient department in the queue.

$$Lq = \frac{\circ^{c+1}}{(c-1)!(c-\circ)^2} * p_0 \qquad (59)$$

**L**: Average number of outpatients in the system = $Lq +$ a/«

**Wq**: Waiting time of outpatients in the queue = Lq/ λ

**W**: Waiting time of outpatients in the system = L/ λ

**Pn** = probability of n outpatients existing in the system.

$$P_n = \begin{cases} \dfrac{(\frac{a}{«})^{n}}{n!} * P_0, & if \;\; 0 < n < C \\ \dfrac{a^{n}}{c!«^{n} c^{n-c}} * P_0, & if \;\; c \le n \end{cases} \qquad (60)$$

Po = Possibility of 0 outpatients existing in the system.

$$p_0 = \left[ \sum_{n=0}^{c-1} \frac{\circ^{n}}{n!} + \frac{\circ^{c}}{c!} \left( \frac{1}{1 - \frac{\circ}{c}} \right) \right]^{-1} \qquad (61)$$

# CHAPTER 4

## RESEARCH FINDINGS AND DISCUSSION

### 4.1 Introduction

In this chapter, we have described how data analysis was done and the findings have been presented. The main results are presented and finally the model is built that is used to respond to the research questions of this study and address the main objectives. The general objective of this project was to apply a queuing model for healthcare services in Muhima District Hospital. In this study we have used a multiple queuing model **M/M/c: FCFS/∞/∞** where we have multiple servers represented by three physicians, infinite system limit and infinite source limit of patients.

### 4.2 The Mean Number of Arrivals per Hour ( a )

**Table 4.1: Mean of arrival interval time**

| | N | Minimum | Maximum | Mean | Std. Deviation |
|---|---|---|---|---|---|
| Arrival interval time | 4662 | 0 | 67 | 4.14 | 6.502 |
| Valid N (listwise) | 4662 | | | | |

The mean number of arrivals has been calculated from the data collected during 36 days of field visit in Muhima hospital in outpatient department. Time duration was recorded for each patient arriving for consultation in outpatient department, then the interval time period separating a patient arrival and the next one was calculated.

Finally the average interval time was calculated. After these calculations we found that every 4.1386 minutes one patient joined the queue. This corresponds to a **=14.4978** patients arrived per hour.

### 4.3 The Mean Number of Patients Served per Hour (μ)

**Table 4.2: Mean of service interval time**

| | N | Minimum | Maximum | Mean | Std. Deviation |
|---|---|---|---|---|---|
| Sevice interval time | 2023 | 0 | 78 | 8.61 | 9.285 |
| Valid N (listwise) | 2023 | | | | |

This mean time of patient served per time period has been calculated based on the records of time a patient enter in the physician's office and the time the patient go out from the office.

These data have been collected in 36 working days and after we have calculated the time spent by each patient in this office. At the end we calculated the average time spent by a patient in the physician's office.

We found that every 8.6134 minutes there was one patient served by all physicians together. This means that every 25.8403 minutes there was one patient served by one physician; this corresponds to $\mu = 2.3220$ patients served per hour per one physician.

## 4.4 System Utilization Factor (ρ).

This factor has been calculated to compare the mean number of arrivals and the mean number of patients served per time period (λ and μ) which can give an idea on the system performance and show that there is a probability that a queue can be formed or not. The system utilization factor ($\rho$) has been calculated using the following formula:

$$\rho = \frac{\lambda}{c\mu} = \frac{14.4978}{3 * 2.3220} = 2.0813 \qquad (62)$$

Here we observe that traffic intensity is greater than one then the queue will grow without bound.

## 4.5 Probability that there is no Outpatient Existing in the System.

$$p_0 = \left[ \sum_{n=0}^{c-1} \frac{\left(\frac{a}{\text{«}}\right)^n}{n!} + \frac{\left(\frac{a}{\text{«}}\right)^c}{c!\left(1 - \frac{a}{c\text{«}}\right)} \right]^{-1} = 0.09849 \qquad (63)$$

**Pn = probability of n outpatients existing in the system.**

$$P_n = \begin{cases} \dfrac{\left(\frac{a}{\text{«}}\right)^n}{n!} * P_0, & if \quad 0 < n < C \\[2ex] \dfrac{a^n}{c!\,\text{«}^n c^{n-c}} * P_0, & if \quad c \le n \end{cases} \qquad (64)$$

After calculations we get the following results

**Table 4.3: Probability of n outpatients existing in the system**

| n | Pn |
|---|---|
| 0 | 0.098488699 |
| 1 | 0.204980855 |
| 2 | 0.213309503 |
| 3 | 0.14798437 |
| 4 | 0.102664783 |
| 5 | 0.071224127 |
| 6 | 0.04941204 |
| 7 | 0.034279811 |
| 8 | 0.023781764 |
| 9 | 0.0164987 |
| 10 | 0.011446043 |

## 4.6 Average Number of Patients Waiting in the Queue before Being Seen by a Physician (Lq).

This is a number of patients that are expected to be on a queue waiting for consultation by a physician. The value has been found by using the following formula:

$$L_q = \frac{\circ^{c+1}}{(c-1)!(c-\circ)^2} * P_0 \qquad (65)$$

After calculations we found **Lq= 7.1129,** this means that at outpatient department of Muhima hospital we can expect to find **7 patients** waiting on the door for a physician.

## 4.7 Average Number of Patients Waiting in the System (L).

The average number of patients waiting in the system represents the number of patients waiting in the queue before being seen by a physician plus the system utilization factor of the system.

$$L = L_q + {}^{a}/_{«} \tag{66}$$

Replacing by respective values we found that:

$$L = 7.1129 + 6.2438 = 13.3566 \tag{67}$$

This means that we can expect 13 patients in the outpatient department including those who are waiting on the queue and those who are being in consultation rooms with physicians.

## 4.8 Average Time a Patient Wait in the Queue before Being Seen by a Physician (Wq).

When a patient arrives at outpatient department can wait for a certain period if all doctors are busy. In this case, the waiting time can vary from a system to another and in our case we found that the waiting time in the queue by using the following formula:

$$W_q = {}^{L_q}/_{a} \tag{68}$$

Replacing with values we found that:

$$W_q = {}^{7.1129}/_{14.4978} = 0.4906 \text{ hours} \approx 29 \text{ min} \tag{69}$$

Here we can see that a patient can wait in the queue around 29 minutes before being seen by a physician.

## 4.9 Average Time a Patient Spends Waiting in the System (W).

This corresponds to the time a patient can spend in the outpatient department since arrival at physician's room up the time he/she come out from the consultation room; this covers the time a patient spends in the queue before being seen by a doctor and the time a patient spends in consultation room with a doctor. The following formula has been used to find the corresponding value:

$$W = L/a \quad (70)$$

By using the values in this formula we found that:

$$W = 13.3566/14.4978 = 0.9213 hours \approx 55.2774 \min \quad (71)$$

Here we can say that the patient can spend 55 minutes in the outpatient department including the time of consultation by a physician.

### 4.10 Correlation Analysis of Waiting Line of Patients

The correlation analysis in this context will measure the association between number of patients' arrival and days of the week. The Figure 3.1 gives an overview on average of variation of arrivals and customers served from Monday to Friday.

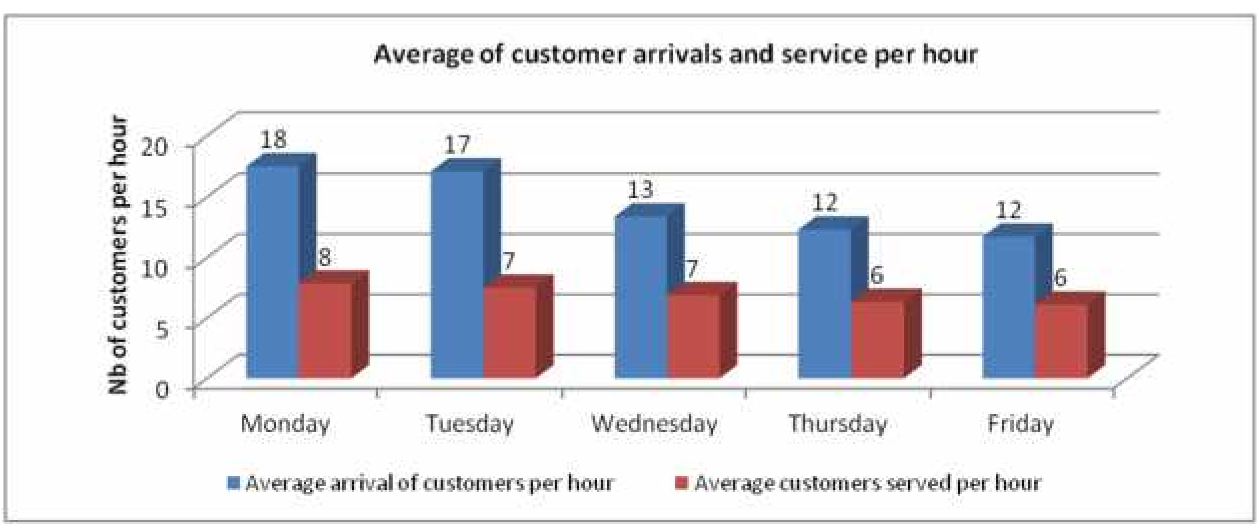

**Figure 3.1: Average of Customer arrivals and service per hour**

From Figure 3.1 we can see that the number of arrivals per hour decline from Monday to Friday 18 patients to 12 patients respectively, while the average number of customers served per hour stay almost stable over the whole week. However we can't conclude on the association based only on this figure, we need to go far and measure this association.

### 4.10.1 Correlation between Days and Arrivals

The correlation between arrivals over the working days of week has been performed using SPSS software under null hypothesis $\mathbf{H_0}$*: There is no correlation between days and customer arrivals,* and the results displayed as follow:

**Table 4.4: Pearson correlation coefficient of Arrivals and working days of the week**

| | | Arrivals | Days |
|---|---|---|---|
| Arrivals | Pearson Correlation | 1 | -0.933* |
| | Sig. (2-tailed) | | 0.021 |
| | N | 5 | 5 |
| Days | Pearson Correlation | -0.933* | 1 |
| | Sig. (2-tailed) | 0.021 | |
| | N | 5 | 5 |

*. Correlation is significant at the 0.05 level (2-tailed).

From the Table 4.4 we find that the Pearson correlation coefficient (r) is -0.933 with small p-value (0.02), therefore we can reject H0; this means that there is a significant negative correlation between days and patient arrivals. In other words there many patients on Monday more than Friday.

### 4.11 Regression Analysis of Patients' Arrivals over the Days of the Week

This regression has been analyzed using two variables X and Y where X represents working days of the week as independent variable and Y stands for number of arrivals of patients as dependent variable.

$$Y = \sim + \quad X \tag{72}$$

The significance of coefficients has been tested at 95% confidence interval under the following null hypothesis:

$H_0$: α=0

$H_0$: β=0

The following results have been found:

**Table 4.5: Significance test of the regression model**

| Model | | Unstandardized Coefficients | | Standardized Coefficients | T | Sig. |
|---|---|---|---|---|---|---|
| | | B | Std. Error | Beta | | |
| 1 | (Constant) | 19.500 | 1.256 | | 15.530 | 0.001 |
| | Days | -1.700 | 0.379 | -0.933 | -4.490 | 0.021 |

a. Dependent Variable: Arrivals

Using the above table we can write the regression model as follow:

$$Y = 19.500 - 1.700X \qquad (73)$$

All constants α and β are statistically significant because p-value are small, 0.001 and 0.021 respectively for α and β; this means that the model is significant. This model can be used for prediction of number of patients expected to come for outpatient consultation from Monday to Friday and from this can be deducted an efficient use of resources.

**4.12 Analysis of the Questionnaire**

The questionnaire was addressed to the nurses and physicians in outpatient department.

The purpose of this questionnaire was to collect additional information about the causes of the queue in their department and proposed solutions to reduce the queue. Using SPSS software we illustrate the composition of our respondents by sex and position in the table below:

**Table 4.6: Composition of respondents by sex and position**

| Category | Staff position | Total |
|---|---|---|

| | | Physician | Nurse | |
|---|---|---|---|---|
| Staff Sex | Male | 6 | 6 | 12 |
| | Female | 0 | 4 | 4 |
| Total | | 6 | 10 | 16 |

We can see that we have interviewed 16 staff include 6 physicians all male and 10 nurses include 4 female. In order to get complementary information guiding to the solutions to reduce the queue, we have asked the nurses and physicians different questions including the following: *"How long do you think a patient can spend waiting before being seen by a physician?"* The answers are summarized in the Figure 8:

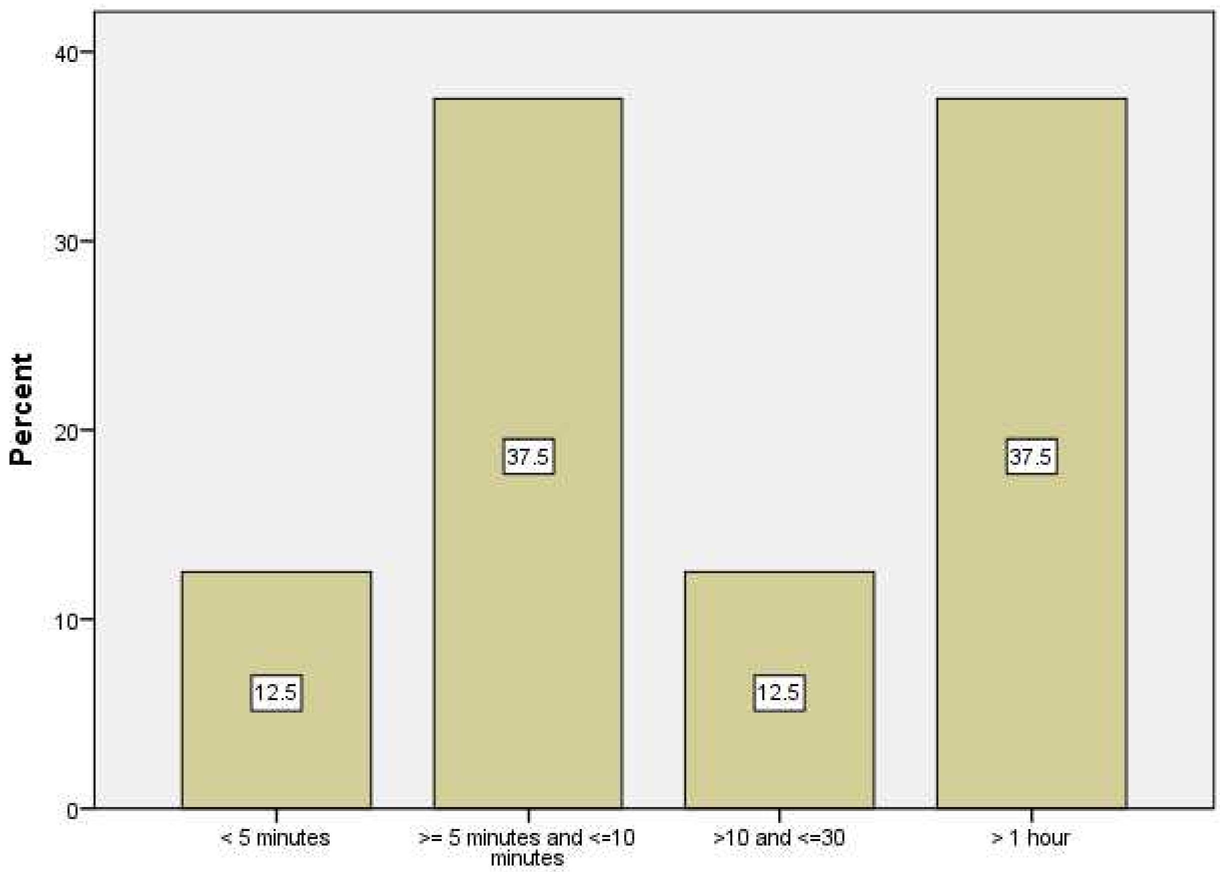


**Figure 3.2: Waiting time of patients in OPD**

Figure 3.2 shows that 37.5% of respondents said that a patient can spend more than one hour waiting before being seen by a physician. We need also to know if the estimated time a patient can spend waiting before being seen by a physician was convenient by using the question: *"Do you think that this time is convenient and comfortable for the patients"*. From Table 8, the answers have been given:

**Table 4.7: Convenience and comfort of patients on waiting Time**

| | Frequency | Percent | Cumulative Percent |
|---|---|---|---|
| Valid No | 16 | 100.0 | 100.0 |
| Yes | 0 | 0 | 100.0 |

Here we can see that all respondents said "No", the waiting time was not convenient and not comfortable for the patients. Then from this, we were curious to know what they think can be causes of this uncomfortable waiting time by asking: *"what are causes of the long waiting time in this department?"* and the answers were as follows:

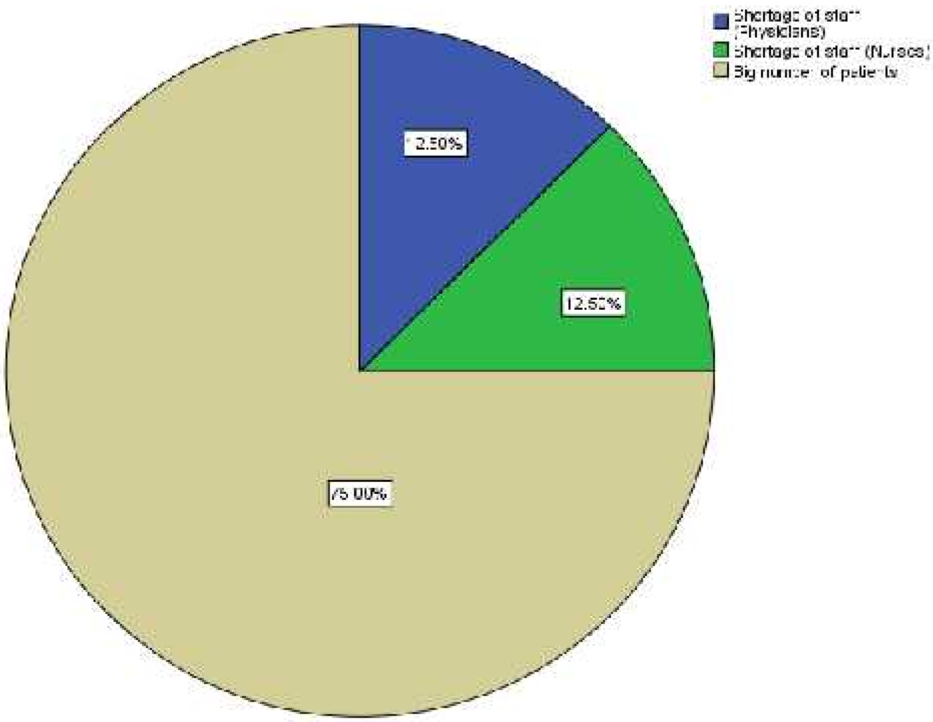


**Figure 3.3: Causes of the long waiting time in OPD**

The interviewed nurses and physicians gave three reasons of the long waiting time in outpatient department where 75% said that the cause is a big number of patients visiting this department while 25% attribute this long waiting time to the shortage of staff. However, this is not enough if we don’t ask the proposed solutions on this issue.

The question was: *“What Solutions do you think can be proposed to reduce the waiting time of patients in this department”*. The interviewed staff proposed the solutions as follow:

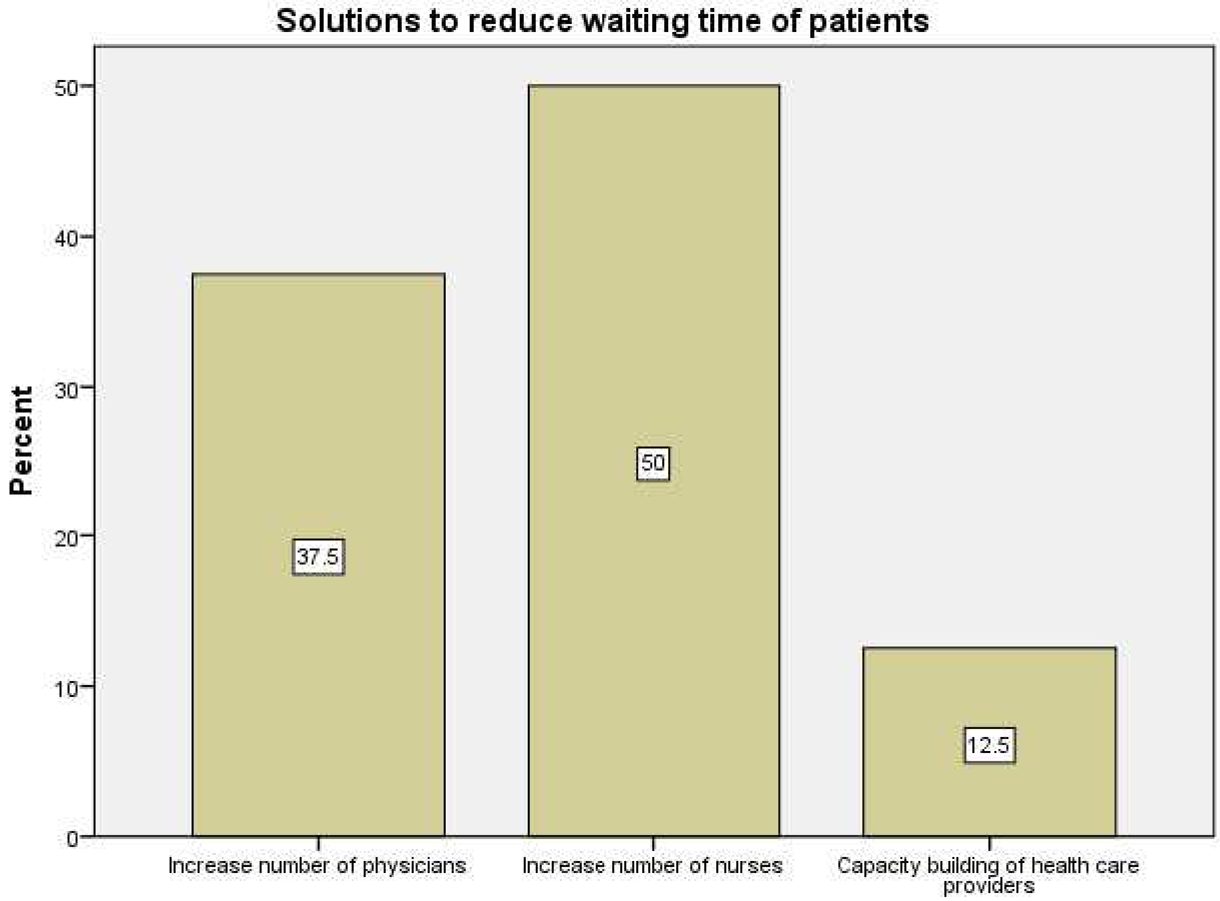


**Figure 3.4: Solutions to reduce waiting time of patients in OPD**

The 37.5% of respondents proposed to increase number of physicians, 50% proposed to increase number of nurses and 12.5% of respondents proposed to strengthen the capacity building of health care providers.

## CHAPTER 5

## SUMMARY, CONCLUSION AND RECOMMENDATIONS

### 5.1 Introduction

This chapter provides summarized results found in our study, give conclusion on the findings and finally the recommendations. To achieve its goal, this chapter will be divided into three subtitles: Summary, Conclusion and recommendations.

### 5.2 Summary

The purpose of my project was to analyze the waiting line of patients and propose solutions for needed resources to reduce the length of queues in Muhima District hospitals and increase patients' satisfaction. The reason for my research is to provide necessary information to policy makers aimed to contribute in wellbeing of population by reducing waiting time for service because in excessive cases, long queues can delay appropriate decision for a specific disease that can cause occurrence of death while patient still wait for service.

The project is composed of five chapters, each of them dealing with different aspects of queuing model for healthcare in public health facilities. Chapter One is introductory and deals with the background of the project, statement of the problem, objectives, research questions, justification and scope of the research.

Chapter Two consists of the literature review of the queuing model for healthcare. The chapter defines basic terminology used in the project and explores the theoretical review of different types of queuing model of interest.

Chapter Three provides the methodological framework of our project. This chapter describes all the methods and instruments used to collect the information we need as well as the procedure of the analysis and interpretation of the information gathered.

Chapter Four concentrates on research findings and discussion on results found after analysis. This chapter shown that system utilization factor is 2.0813, this explains that the queue will grow without bound. The average number of patients waiting in the system is 13 patients. Here we have also seen that a patient can wait in the queue around 29 minutes before being seen by a physician and 55 minutes in the system. The correlation analysis has revealed a significant negative correlation between days and patient arrivals (-0.933) which means that there are many patients on Monday more than Friday.

The interviewed nurses and physicians gave three reasons of the long waiting time in outpatient department where 75% said that the cause is a big number of patients visiting this department while 25% attribute this long waiting time to the shortage of staff.

To reduce the waiting time, 37.5% of respondents proposed to increase number of physicians, 50% proposed to increase number of nurses and 12.5% of respondents proposed to strengthen the capacity building of health care providers.

Conclusions are drawn in Chapter Five. The main aim of the project is to evaluate and predict the system performance for Muhima District hospital and propose solutions on needed resources to improve the quality of service offered to the patients visiting this hospital and it has been reached. We suggest that the number of physicians and nurses be increased and a staffing plan to be developed in the hospitals in order to manage an efficient shifting which can provide more efforts at the beginning of the week.

### 5.3 Conclusions

The main objective of this project was to apply a queuing model for healthcare services in Muhima District Hospital.

The findings show that the system utilization factor is greater than one this explains that the queue will grow without bound. There were a big number of patients waiting in the queue and they waited for a long time before being seen by a physician. The correlation analysis has revealed a significant negative correlation between days and patient arrivals which means that there are many patients on Monday more than Friday.

The reasons given by 75% of staff interviewed were the big number of patients visiting this department while 25% attribute this long waiting time to the shortage of staff.

To reduce the waiting time, 37.5% of respondents proposed to increase number of physicians, 50% proposed to increase number of nurses and 12.5% of respondents proposed to strengthen the capacity building of health care providers. The hospital should develop a staffing plan and put more effort in the beginning of the week for efficient use of available resources.

## 5.4 Recommendations

### 5.4.1 To increase number of staff in outpatient department

There is need to increase number of physicians and nurses in Outpatient department in order to respond to the needs of a big number of patients visiting this department. The increase of staff will also need to increase the number of consultation rooms to provide a place to be used by additional staff.

### 5.4.2 To strengthen the capacity building of health care providers.

The strengthening of capacity building of staff is necessary for health provider's staff working in Outpatient department of Muhima hospital to give a quick service to the patient.

The capacity building strengthening is also needed to the nurses working in Health Centers of Muhima Hospital's catchment area so that many diseases can be treated at low level and the number of transfers from Health center to the hospital will decrease.

### 5.4.3 Develop a staffing plan that put more effort in the beginning of the week

The staffing plan should be developed depending on the trend of patients during the working days. This means that the Outpatient department should provide a big number of nurses and physicians in the beginning of the week in order to respond to the big number of patients coming in these days. The consultation in Outpatient department should also begin in the morning as early as possible so that the patients who are coming in the morning can be served without spending a long time on the queue.

### 5.4.4. Recommendation for further research

Other researchers should go forward and conduct the same study in other departments in order to estimate the total time a patient can spend in the hospital considering the whole chain he/she has to pass through.

**REFERENCES**


Allen, A.O. (1978) *Probability, Statistics and Queuing Theory: With Computer Applications*, Academic Press, Florida.

Asmussen, S. (2003). *Applied probability and queues* (2nd ed). New York: Springer.

Baccelli, F., & Brenaud, P. (2003). *Elements of Queuing Theory: Palm martingale calculus and stochastic recurrences* (2nd ed). New York: Springer.

Barbeau, Michel; Kranakis, Evangelos (2007). *Principles of Ad-hoc Networking*. John Wiley & Sons Ltd. New Jersey.

Capasso, V. (2008). *An introduction to continous Tme stochastic processes: Theory, models, and Application to Finance, Biology and Medecine.* Birkäuser, New York.

Carl-Erik. (2003). *Model assisted survey sampling*. Springer. pp. New Mexico.

Davis, M. M. (2003). *Fundamentals of Operations Management.* Boston: McGraw- Hill Irwin, Fourth Edition.

Depelteau, F. (2000). *The approach of a research in the human sciences*. Brussels, Rue des Minimes

Giambene, G. (2005). *Queuing theory and telecommunications: Network and Applications*. New York: Springer.

Gorard, S. (2013). *Research Design: Robust approaches for the social sciences,* London, 218.

Hammersley, M. (2002). *Educational research*. Vol. 1, Paul Chapman Publishing Ltd, London

Medhi, J. (2002). Stochastic Models in Queuing Theory. (2nd ed). Academic Press. India

Petit Larousse, (2008). Paris.

Sundarapandian, V. (2009). *Queuing Theory. Probability, Statistics and Queuing Theory*. PHI Learning, First edition. New Delhi.

Sztrik, J. (2000). Finite-source queuing systems and their applications: *University of Debrecen, Debrecen, Hungary. J*. No. 21.

Taha, A. H. (2005). *Operation Research: An Introduction*. Delhi: Pearson Education, Inc.,Seven Edition. New Delhi.

Cochran, K.J. (2006). A Multi-stage Stochastic Methodology for Whole Hospital Bed Planning Under Peak Loading. *International Journal of Industrial and Systems Engineering*, 8-35.

Hall, W. R. (2006), *Patient Flow: The New Queuing Theory for Healthcare*. OR/Ms, Today. California.

Kandemir - Caues, C., Cauas, L. (2007). An Application of Queuing Theory to the Relationship between Insulin Level and Number of Insulin Receptors. *Turkish Journal of Biochemistry*, 32-38.

Nosek, A.R., Wislon, P. J. (2001). Queuing Theory and Customer Satisfaction: *A Review of Terminology, Trends and Applications to Pharmacy Practice, Hospital Pharmacy*, 275-279.

Obamiro, J. (2010). Queuing Theory and Patient Satisfaction: An Overview of Terminology and Application in Ante-Natal Care Unit. *Petroleum-Gas University of Ploiesti bulletin.*Vol. 62 No.1.

Schlechter, (2009). Hershey Medical Center to open redesigned emergency room. *The Patriot-News.*

Singh, V. (2011). Use of Queuing Models in Health Care, Department of Health Policy and Management, University of Arkanses for medical science. *International Journal of Computing and Business Research*. p 1-2.

Puhalskii, Anatolii A. and Josh E. Reed. (2010). On many-server queues in heavy traffic. *The Annals of Applied Probability*. Volume 20, No. 1 (2010), 129-195.

## APPENDICES

### *Appendix I*: Research Questionnaire

This questionnaire is used to collect information from physicians and nurses working in outpatient department at Muhima district hospital.

*N.B:* ***The information that will be given is only for academic purpose and will be kept confidentially.***

1. Sex
    a. Male ☐
    b. Female ☐
2. What is your position?
    a. Physician ☐
    b. Nurse ☐
    c. Other ☐
    (Specify)……………………………………………………………………………
3. How long do you think a patient can spend waiting before being seen by a physician? *(Select only one answer)*
    a. Less than 5 minutes ☐
    b. Between 5 minutes and 10 minutes ☐
    c. Greater than 10 minutes and 30 minutes ☐
    d. Greater than 30 minutes and 1 hour ☐
    e. More than 1 hour *(Specify time in hours)*………………….. ☐

4. Do you think that this time is convenient and comfortable for the patients?
    a. Yes ☐
    b. No ☐
5. If No, what are causes of the long waiting time in this department? *(Many answer can be selected)*
    a. Shortage of staff (physicians) ☐
    b. Shortage of staff (nurses) ☐
    c. Big number of patients ☐
    d. Delay of personnel in service delivery ☐
    e. Shartage of consultation rooms (Infrastructure issue). ☐
    f. Other*(Specify)*………………………………………………………………… ☐
6. What solutions do you think can be proposed to reduce the waiting time of patients in this department? *(Many answers can be selected)*
    a. Increase number of physicians in outpatient department ☐
    b. Increase number of nurses in outpatient department ☐
    c. Increase number of outpatient consultation rooms ☐
    d. Capacity building of health care providers. ☐
    e. Reduce the number of steps where a patient has to pass before being seen by a physician. ☐
    f. Other *(Specify)* ☐

    ………………………………………………………………………………………

    ………………………………………………………………………………………

**Thanks for your kind participation.**

*Appendix II:* **Customer Arrival and Service Tool**

| | | | | | **Customer service** | |
|---|---|---|---|---|---|---|
| **N⁰** | **Date (dd/mm/yyyy)** | **Day** | **Time(hh:min:ss)** | **Customer arrival (I)** | **In (I)** | **Out (I)** |
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| ⋮ | | | ⋮ | ⋮ | ⋮ | ⋮ |

***Appendix III:* Answers of the Questionnaire:**

**Waiting time of a patient**

| | | Frequency | Percent | Cumulative Percent |
|---|---|---|---|---|
| Valid | < 5 minutes | 2 | 12.5 | 12.5 |
| | >= 5 minutes and <=10 minutes | 6 | 37.5 | 50.0 |
| | >10 and <=30 | 2 | 12.5 | 62.5 |
| | > 1 hour | 6 | 37.5 | 100.0 |
| | Total | 16 | 100.0 | |

**Causes of the queue**

| | | Frequency | Percent | Cumulative Percent |
|---|---|---|---|---|
| Valid | Shortage of staff (Physicians) | 2 | 12.5 | 12.5 |
| | Shortage of staff (Nurses) | 2 | 12.5 | 25.0 |
| | Big number of patients | 12 | 75.0 | 100.0 |
| | Total | 16 | 100.0 | |

**Solutions to reduce waiting time of patients**

| | | Frequency | Percent | Cumulative Percent |
|---|---|---|---|---|
| Valid | Increase number of physicians | 6 | 37.5 | 37.5 |
| | Increase number of nurses | 8 | 50.0 | 87.5 |
| | Capacity building of health care providers | 2 | 12.5 | 100.0 |
| | Total | 16 | 100.0 | |

*Appendix IV:* **Probability of n Outpatients Existing in the System ($P_n$).**

| **N** | **Pn** |
|---|---|
| 0 | 0.098488699 |
| 1 | 0.204980855 |
| 2 | 0.213309503 |
| 3 | 0.14798437 |
| 4 | 0.102664783 |
| 5 | 0.071224127 |
| 6 | 0.04941204 |
| 7 | 0.034279811 |
| 8 | 0.023781764 |
| 9 | 0.0164987 |
| 10 | 0.011446043 |
| 11 | 0.00794074 |
| 12 | 0.005508922 |
| 13 | 0.003821838 |
| 14 | 0.002651416 |
| 15 | 0.001839431 |
| 16 | 0.001276113 |
| 17 | 0.000885309 |
| 18 | 0.000614187 |
| 19 | 0.000426095 |

| | |
|---|---|
| 20 | 0.000295605 |
| 21 | 0.000205077 |
| 22 | 0.000142273 |
| 23 | 9.87E-05 |
| 24 | 6.85E-05 |
| 25 | 4.75E-05 |
| 26 | 3.30E-05 |
| 27 | 2.29E-05 |
| 28 | 1.59E-05 |
| 29 | 1.10E-05 |
| 30 | 7.63E-06 |